Microscopic calculations of elastic scattering α+n, α+p, $\alpha+{}^3\mathrm{He}$ and $\alpha+\alpha$ from a realistic nucleon-nucleon interaction

Jérémy Dohet-Eraly

Physique Nucléaire et Physique Quantique (P.N.P.Q.)
Université Libre de Bruxelles (ULB)
Research Fellow of F.R.S.-FNRS

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- We would like to obtain a predictive model... but the way to reach it is still long.

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	Effective	Realistic
Calculations	analytically	numerical,heavy
Agreement with experiment	good	α+nucleon less good
Fit	Yes	"No"

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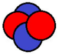

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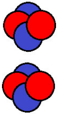

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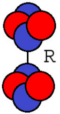

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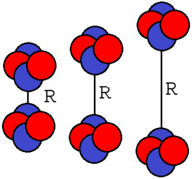

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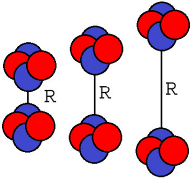

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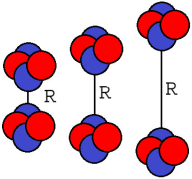

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- Solution : the Microscopic R-Matrix [Baye and Heenen, Nucl. Phys. A 233 (1974) 304].

R-matrix

Internal region

Microscopic description

$$\Psi_{int} = \sum_R f_R^{J\pi} \psi^{JM\pi}(R)$$

External region

Macroscopic description

$$\Psi_{ext} = \phi_1\phi_2 \left(\cos(\delta_l)F_l(k\rho) + \sin(\delta_l)G_l(k\rho)\right)$$

Antisymmetrization neglected

0 a ρ

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- To solve this problem, we have to make more complex the cluster. One solution is the Unitary Correlation Operator Method [Feldmeier, Neff, Roth and Schnack, Nucl. Phys. A 632 (1998) 61, Neff and Feldmeier, Nucl. Phys. A 713 (2003) 311].

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$$C_r = \exp\left[-i\sum_{i<j}^{A}\sum_{ST}\frac{1}{2}\left[s_{ST}(r_{ij})p_{r_{ij}} + p_{r_{ij}}s_{ST}(r_{ij})\right]\Pi_{ST}\right].$$

$$C_\Omega = \exp\left[-i\sum_{i<j}^{A}\sum_{T}\vartheta_T(r_{ij})s_{12}(\mathbf{r}_{ij},\mathbf{p}_{\Omega_{ij}})\Pi_T\right].$$

Let a two-nucleon state be

$$\phi_1(\mathbf{r_1})\phi_2(\mathbf{r_2}) = \phi_{rel}(\mathbf{r}_{12})\phi_{cm}(\mathbf{r}_{cm})$$

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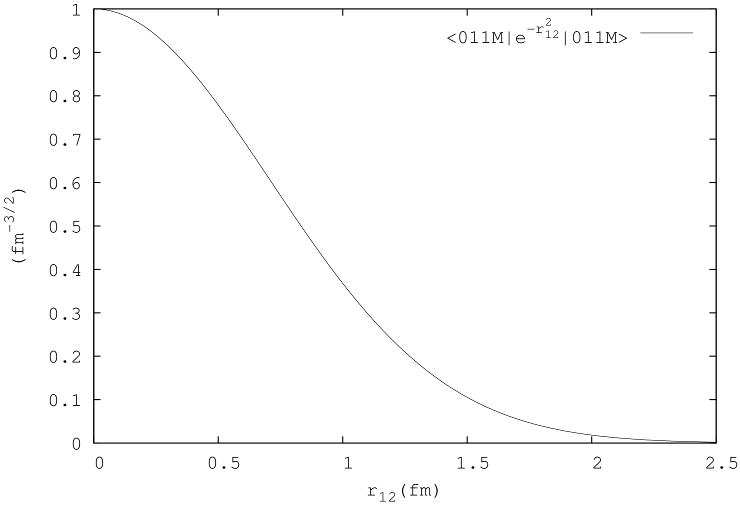
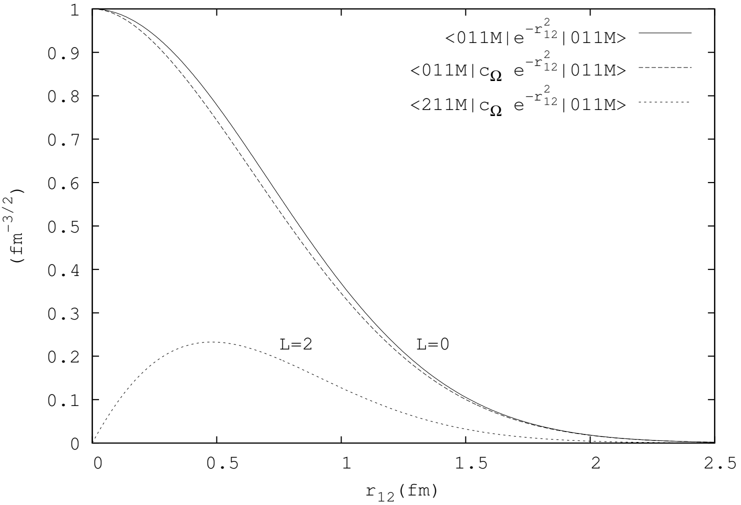


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.5
1
1.5
2
2.5
<011M|e^{-r_12^2}|011M>
(fm^{-3/2})
r_12(fm)



<011M|e^{-r_{12}^2}|011M>
<011M|c_Ω e^{-r_{12}^2}|011M>
<211M|c_Ω e^{-r_{12}^2}|011M>
L=2
L=0
(fm^{-3/2})
r_{12}(fm)
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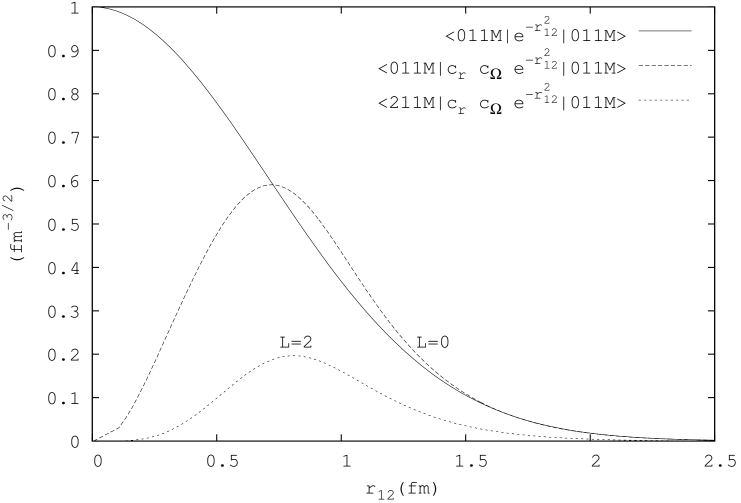
0.4
0.3
0.2
0.1
0
0.5
1
1.5
2
2.5



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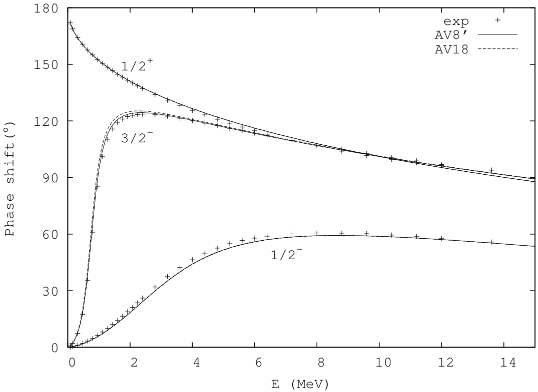
$\alpha + \mathrm{n}$



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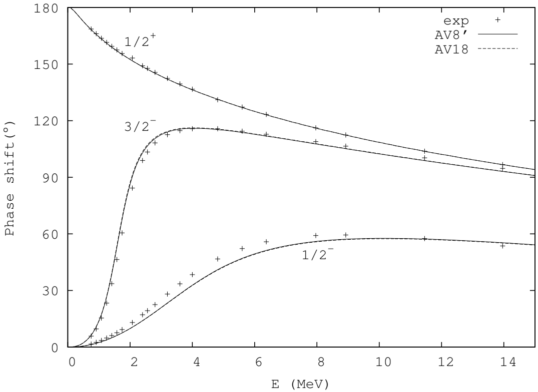
$\alpha + \mathrm{p}$



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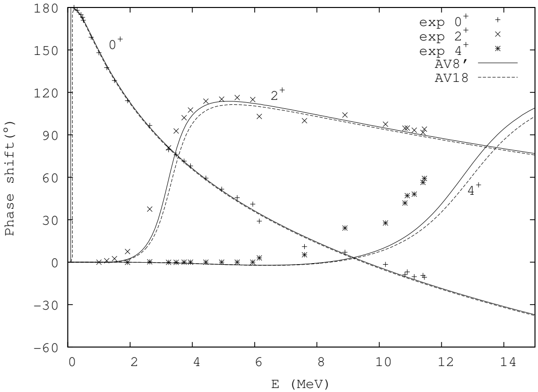
$\alpha + \alpha$



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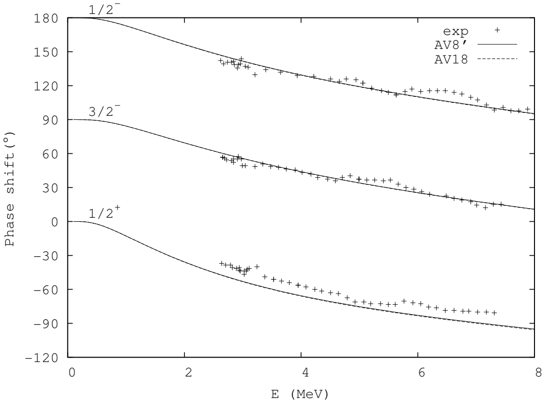
$\alpha + {}^3$ He



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- The final aim is to take account of three-body interactions and to make more complex the wave function while keeping the good agreement without fit.

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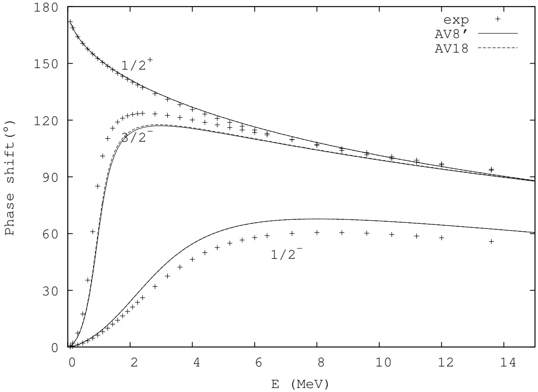
Thank you for your attention!



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